Lecture 4: Dynamics of small open economies cont'd.

Open economy macroeconomics, Fall 2006

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From last week's lecture: A stochastic small open economy model

- Model ingredients:
  - Asset structure: riskless bond which pays a constant real interest rate r
  - Preferences:

$$U_t = E_t\left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s)\right]$$

    where

$$u(C) = C - \frac{a_0}{2}C^2, \qquad a_0 > 0$$

    and

$$\beta = \frac{1}{1+r}$$

  - Source of uncertainty: endowment follows a stochastic process

- Question: what are the implications for the current account of a positive innovation to output in period t ($\varepsilon_t > 0$)?

- General recipe for getting the answer:

  1. State the period s budget constraint

  2. Iterate on the period t budget constraint to derive the intertemporal budget constraint (remember transversality condition)

  3. State the representative household's optimisation problem and derive consumption Euler equation

     (a) Solve out for consumption from the period s budget constraint

     (b) Substitute into expression for lifetime utility

     (c) Find the first-order condition with respect to bond holdings in period $t+1$

4. Combine the consumption Euler equation and the intertemporal budget constraint to derive expression for consumption in period t as a function of expected future endowments

5. Derive the closed form for consumption

   (a) Use endowment process to get expression for expected future endowments as a function of the endowment in period t

   (b) Substitute into expression for period t consumption

6. Use the definition of the current account to express the current account as a function of the endowment in period t

7. Compute impulse responses of the current account to an innovation to output in period t.

- Example 1: Endowment follows first-order autoregressive process

$$Y_t - \overline{Y} = \rho \left(Y_{t-1} - \overline{Y} \right) + \varepsilon_t$$

where $0 \leq \rho \leq 1$ and $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon_s] = 0$ for $s \neq$ t

1. State the period s budget constraint

$$B_{s+1} = (1 + r)B_s + Y_s - C_s$$

2. Iterate on the period t budget constraint to derive the intertemporal budget constraint (remember transversality condition)

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1 + r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

3. State the representative household's optimisation problem and derive the consumption Euler equation

   (a) Solve out for consumption from the period s budget constraint

$$C_s = (1+r)B_s + Y_s - B_{s+1}$$

   (b) Substitute into expression for lifetime utility

$$\begin{aligned} U_t &= E_t\left[\sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s + Y_s - B_{s+1})\right] \\ &= u((1+r)B_t + Y_t - B_{t+1}) + \beta E_t\left[u((1+r)B_{t+1} + Y_{t+1} - B_{t+2})\right] \\ &\quad + \ldots \end{aligned}$$

   (c) Find the first-order condition with respect to bond holdings in period $t+1$

$$\begin{aligned} u'(C_t) &= \beta(1+r)E_t\left[u'(C_{t+1})\right] \\ C_t &= E_t\left[C_{t+1}\right] \\ E_t\left[C_s\right] &= C_t \end{aligned}$$

4. Combine the consumption Euler equation and the intertemporal budget constraint to derive expression for consumption in period t as a function of expected future endowments

$$
\begin{aligned}
\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[C_s\right] &= (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right] \\
C_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} &= (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right] \\
C_t \frac{1}{1-\frac{1}{1+r}} &= (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right] \\
C_t &= rB_t + \underbrace{\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right]}_{E_t\widetilde{Y}_t}
\end{aligned}
$$

5. Derive the closed form for consumption

(a) Use endowment process to get expression for expected future endowments as a function of the endowment in period t

$$
\begin{aligned}
Y_t - \overline{Y} &= \rho\left(Y_{t-1} - \overline{Y}\right) + \varepsilon_t \\
E_t\left[Y_{t+1} - \overline{Y}\right] &= \rho\left(Y_t - \overline{Y}\right) \\
E_t\left[Y_{t+2} - \overline{Y}\right] &= \rho E_t\left[Y_{t+1} - \overline{Y}\right] = \rho^2\left(Y_t - \overline{Y}\right) \\
&\vdots \\
E_t\left[Y_s - \overline{Y}\right] &= \rho^{s-t}\left(Y_t - \overline{Y}\right)
\end{aligned}
$$

(b) Substitute into expression for period t consumption

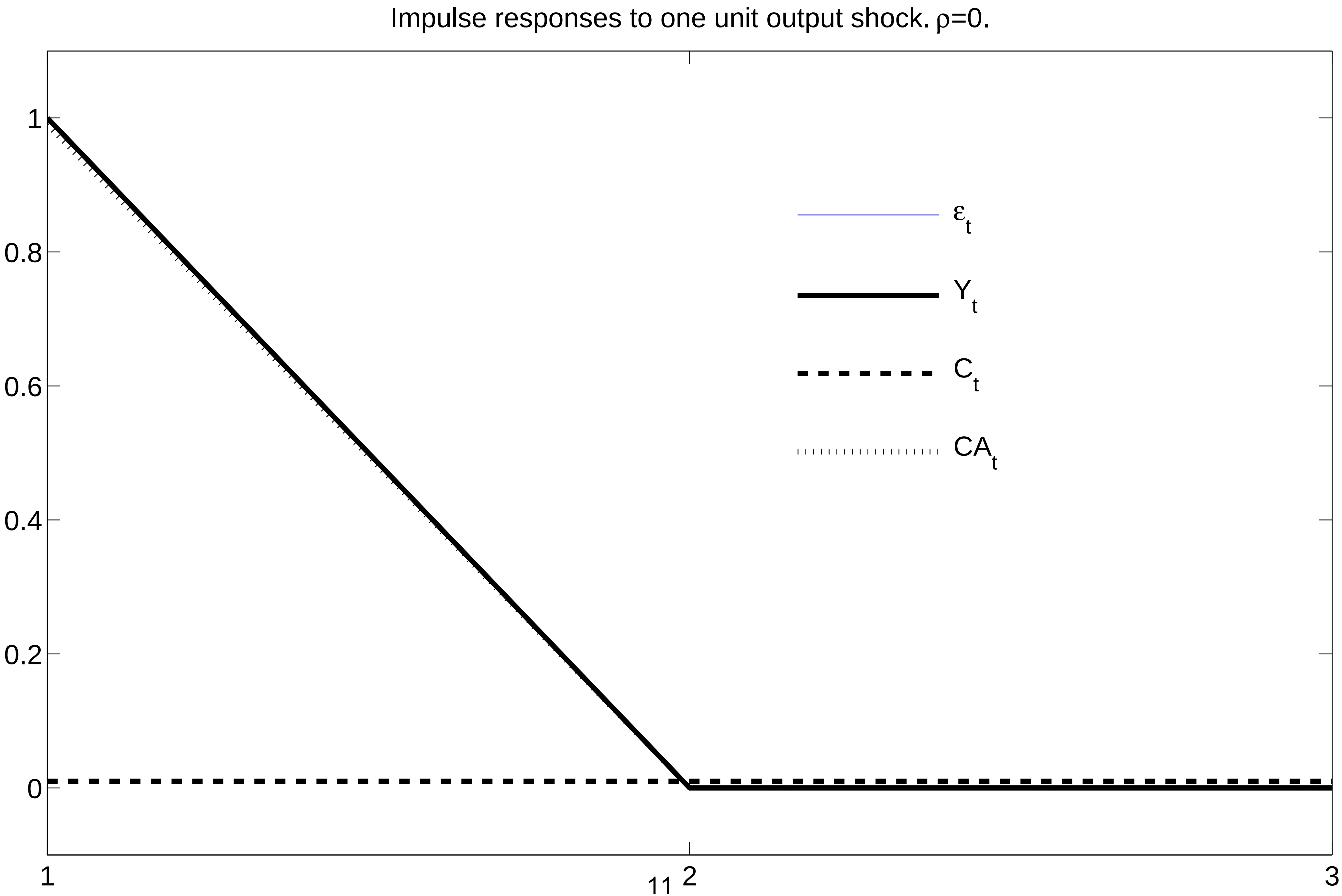
$$
\begin{aligned}
C_t &= rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right] \\
&= rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \left(\rho^{s-t}\left(Y_t - \overline{Y}\right) + \overline{Y}\right) \\
&= rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{\rho}{1+r}\right)^{s-t} \left(Y_t - \overline{Y}\right) + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \overline{Y} \\
&= rB_t + \overline{Y} + \frac{r}{1+r-\rho}\left(Y_t - \overline{Y}\right) \\
&= rB_t + \overline{Y} + \frac{r\rho}{1+r-\rho}\left(Y_{t-1} - \overline{Y}\right) + \frac{r}{1+r-\rho}\varepsilon_t
\end{aligned}
$$

6. Use the definition of the current account to express the current account as a function of the endowment in period t

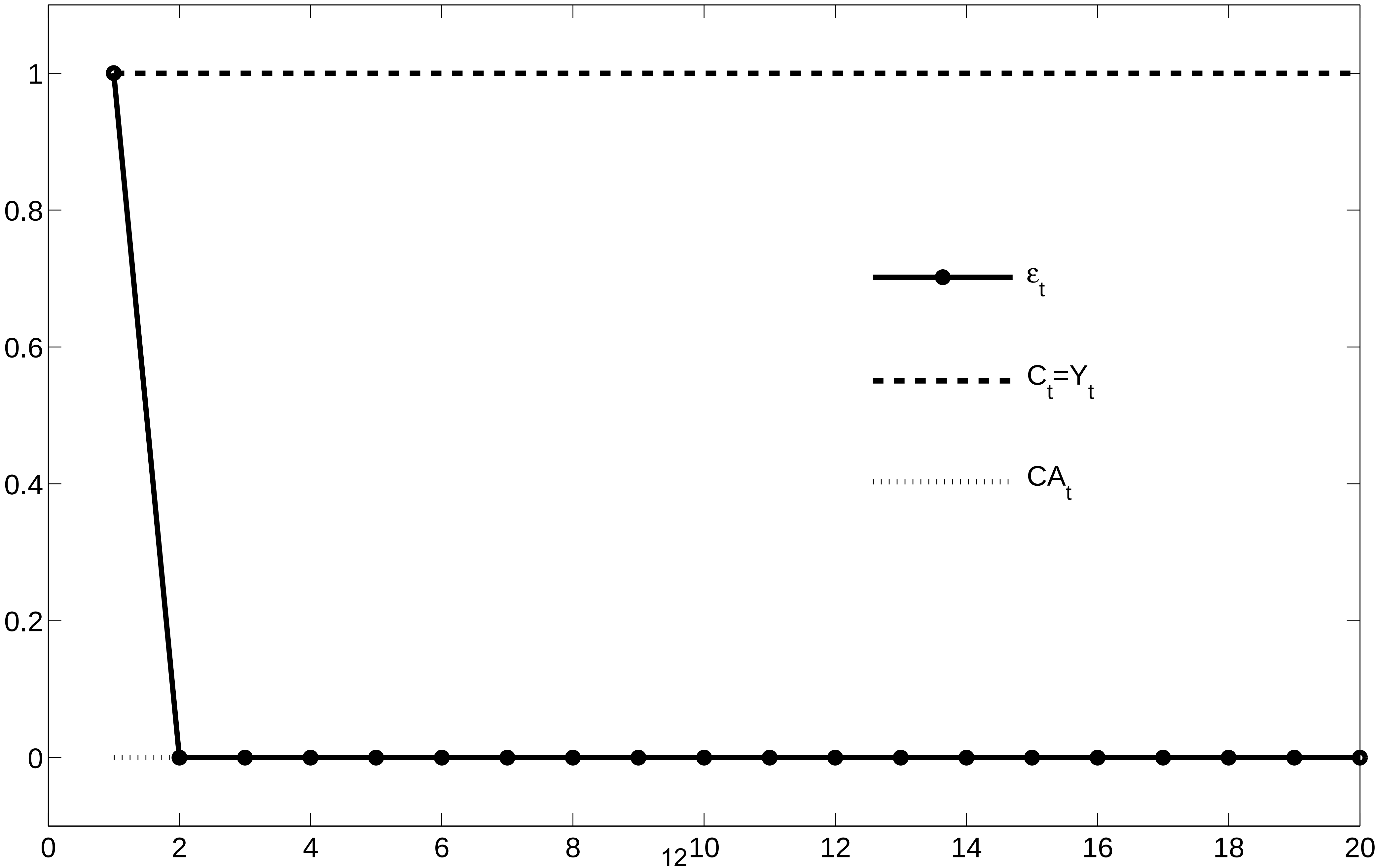
$$\begin{aligned} CA_t &= rB_t + Y_t - C_t \\ &= rB_t + Y_t - \left(rB_t + \overline{Y} + \frac{r}{1+r-\rho}\left(Y_t - \overline{Y}\right)\right) \\ &= \frac{1-\rho}{1+r-\rho}\left(Y_t - \overline{Y}\right) \end{aligned}$$

7. Compute impulse responses of the current account to an innovation to output in period t.

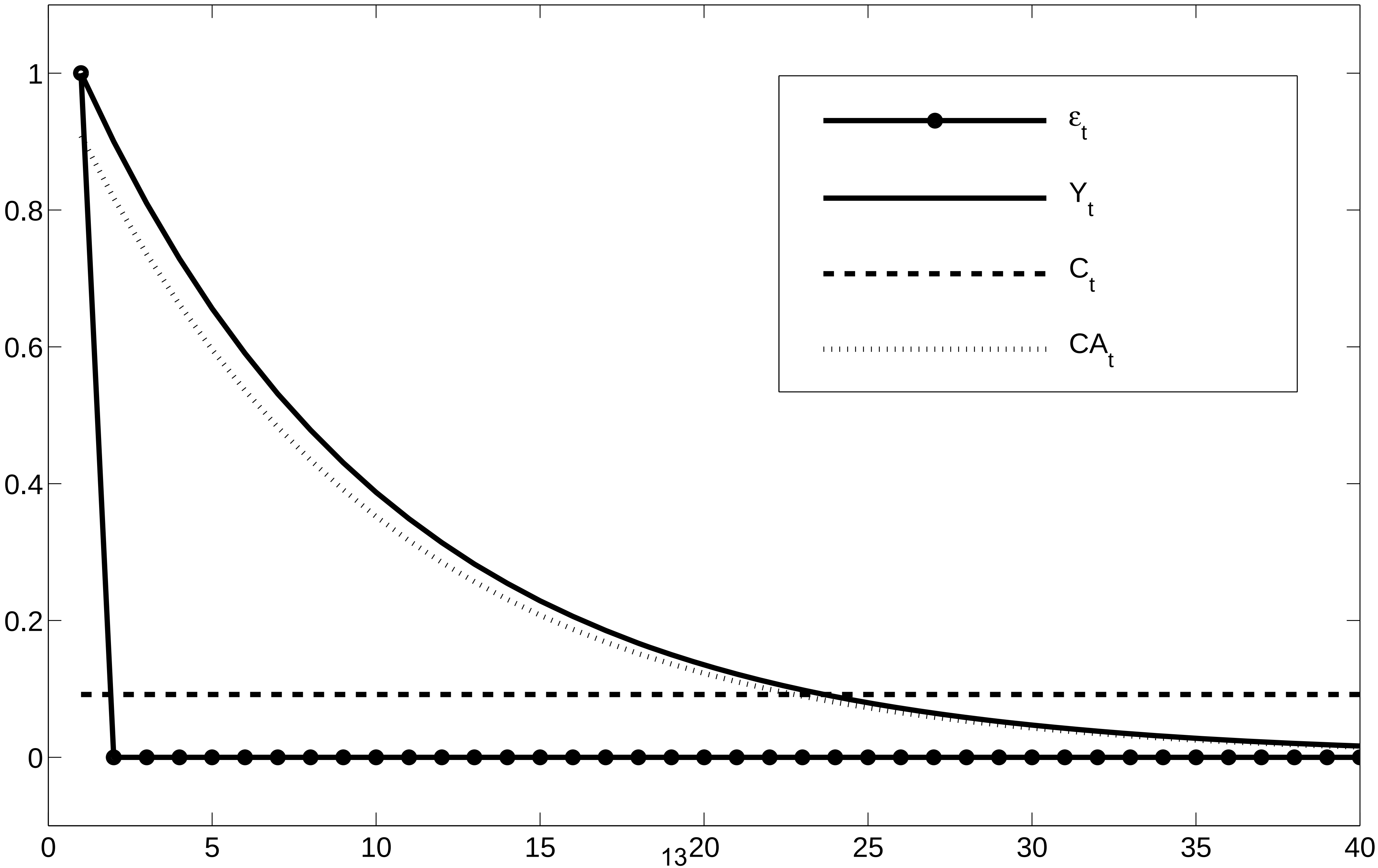
$$\begin{aligned} \frac{\partial CA_t}{\partial \varepsilon_t} &= \frac{1-\rho}{1+r-\rho}, \frac{\partial CA_{t+1}}{\partial \varepsilon_t} = \frac{1-\rho}{1+r-\rho}\rho, \frac{\partial CA_{t+2}}{\partial \varepsilon_t} = \frac{1-\rho}{1+r-\rho}\rho^2 \\ &\dots \\ \frac{\partial CA_s}{\partial \varepsilon_t} &= \frac{1-\rho}{1+r-\rho}\rho^{s-t} \end{aligned}$$


Impulse responses to one unit output shock. ρ=0.
1
0.8
0.6
0.4
0.2
0
1
2
3
ε_t
Y_t
C_t
CA_t

Impulse responses to one unit output shock. ρ=1.



Impulse responses to one unit shock to output. ρ=0.9.



```
clear all;
%Set parameters
P = 40; %Impulse response horizons
beta = 0.99; %Discount factor
r = (1-beta)/beta; %Exogenous real interest rate
YMEAN = 0; % Mean output level
rho = 0.9; % Persistence of output process
B(1) = 0; % Initial level of debt

%Shock process (one unit shock in period 1)
e = zeros(1,P);
e(1)=1;

%Output process (assuming Y(0)=YMEAN)
Y(1) = YMEAN+e(1);
for s = 2:P;
Y(s)=rho*(Y(s-1)-YMEAN)+YMEAN+e(s);
end


%Solution for consumption, net foreign assets and current account balance
for s = 1:P;
C(s) = r*B(s)+YMEAN+(r/(1+r-rho))*(Y(s)-YMEAN); %Consumption function
B(s+1)=(1+r)*B(s)+Y(s)-C(s); % Period s budget constraint
CA(s) = B(s+1)-B(s); % Definition of current account balance
end
```

- Example 2 (exercise 4 to chapter 2 in Obstfeld&Rogoff)

- Endowment process

$$Y_{t+1} - Y_t = \rho\left(Y_t - Y_{t-1}\right) + \varepsilon_{t+1}$$

  where $0 \leq \rho \leq 1$ and $E\left[\varepsilon_t\right] = 0$ and $E\left[\varepsilon_t \varepsilon_s\right] = 0$ for $s \neq t$.

- What are the current account implications of a positive innovation to the endowment in period $t+1$ ($\varepsilon_{t+1} > 0$).

- (1)-(3) as before

4. Combine the consumption Euler equation and the intertemporal budget constraint to derive expression for consumption in period t as a function of expected future endowments

$$C_t = rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right]$$

Trick! Express the change in consumption as

$$C_{t+1} - C_t = \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} \left(E_{t+1} - E_t\right) Y_s$$

where $\left(E_{t+1} - E_t\right) Y_s$ denotes the revision of expectations of Y_s due to information that arrives between t and $t+1$

$$
\begin{aligned}
C_{t+1}-C_t &= rB_{t+1}+\frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} E_{t+1}\left[Y_s\right] - rB_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right] \\
&= r\left(B_{t+1}-B_t\right)+\frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} E_{t+1}Y_s - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_tY_s \\
&= r\underbrace{\left(rB_t+Y_t-rB_t-\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\left[Y_s\right]\right)}_{=CA_t=rB_t+Y_t-C_t} \\
&\quad +\frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} E_{t+1}Y_s - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_tY_s \\
&= rY_t - r\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_tY_s + \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} E_{t+1}Y_s \\
&= rY_t - rY_t - \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} E_tY_s + \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} E_{t+1}Y_s \\
&= \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t+1)} \left(E_{t+1}-E_t\right)Y_s
\end{aligned}
$$

5. Derive the closed form for consumption

(a) Use endowment process to get expression for expected future endowments as a function of the output innovation ε_{t+1}

$$\begin{aligned} E_{t+1}Y_{t+1} &= Y_{t+1} = Y_t + \rho(Y_t - Y_{t-1}) + \varepsilon_{t+1} \\ E_tY_{t+1} &= Y_t + \rho(Y_t - Y_{t-1}) \end{aligned}$$

implies

$$(E_{t+1} - E_t)\, Y_{t+1} = \varepsilon_{t+1}$$

$$\begin{aligned} E_{t+1}Y_{t+2} &= Y_{t+1} + \rho(Y_{t+1} - Y_t) \\ E_tY_{t+2} &= E_tY_{t+1} + \rho(E_tY_{t+1} - Y_t) \end{aligned}$$

implies

$$(E_{t+1} - E_t)\, Y_{t+2} = 1 + \rho)(E_{t+1} - E_t)Y_{t+1} = (1 + \rho)\varepsilon_{t+1}$$

$$\begin{aligned} E_{t+1}Y_{t+3} &= E_{t+1}Y_{t+2} + \rho(E_{t+1}Y_{t+2} - Y_{t+1}) \\ E_tY_{t+3} &= E_tY_{t+2} + \rho(E_tY_{t+2} - E_tY_{t+1}) \end{aligned}$$

implies

$$
\begin{aligned}
(E_{t+1} - E_t)\, Y_{t+3} &= (1+\rho)\,(E_{t+1} - E_t)\, Y_{t+2} - \rho\,(E_{t+1} - E_t)\, Y_{t+1} \\
&= (1+\rho)(1+\rho)\varepsilon_{t+1} - \rho\varepsilon_{t+1} \\
&= \left(\rho^2 + \rho + 1\right)\varepsilon_{t+1}
\end{aligned}
$$

implies

$$
(E_{t+1} - E_t)\, Y_s = \left(1 + \rho + \rho^2 + \cdots + \rho^{s-(t+1)}\right)\varepsilon_{t+1} = \frac{1-\rho^{s-t}}{1-\rho}\varepsilon_{t+1}
$$

where we have used the fact that for $k < 1$, $\sum_{i=0}^{m} k^i = \frac{1-k^{m+1}}{1-k}$

(b) Substitute into expression for period t consumption

$$
\begin{aligned}
C_{t+1} - C_t &= \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-(t+1)} (E_{t+1} - E_t) Y_s \\
& \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-(t+1)} \left(\frac{1-\rho^{s-t}}{1-\rho} \right) \varepsilon_{t+1} \\
&= \frac{1}{1-\rho} \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{s-(t+1)} \\
& - \frac{\rho}{1-\rho} \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{\rho}{1+r} \right)^{s-(t+1)} \\
&= \frac{1}{1-\rho} \frac{r}{1+r} \left(\frac{1+r}{r} - \frac{\rho(1+r)}{1+r-\rho} \right) \varepsilon_{t+1} \\
&= \frac{1+r}{1+r-\rho} \varepsilon_{t+1} \\
&= \frac{1+r}{1+r-\rho} (E_{t+1} - E_t) Y_{t+1}
\end{aligned}
$$

6. Use the definition of the current account to express the current account as a function of the output innovation ε_{t+1}

$$
\begin{aligned}
CA_{t+1} - E_t CA_{t+1} &= rB_{t+1} + Y_{t+1} - C_{t+1} - rB_{t+1} - E_t Y_{t+1} + E_t C_{t+1} \\
&= (Y_{t+1} - E_t Y_{t+1}) - (C_{t+1} - E_t C_{t+1}) \\
&= \varepsilon_{t+1} - \frac{1+r}{1+r-\rho}\varepsilon_{t+1} \\
&= \frac{-\rho}{1+r-\rho}\varepsilon_{t+1}
\end{aligned}
$$

- A positive output innovation now implies a current account deficit
- Permanent income fluctuates more than current income $\longrightarrow$ consumption increases more than current output
- Deaton's paradox: In the data, output growth is positively serially correlated, yet consumption does not respond more than proportionally to output changes.

Adding investment to the stochastic infinite-horizon small open economy model

- Production function

$$Y_t = A_t F(K_t)$$

where A_t is a random variable

- Capital accumulation (no depreciation)

$$K_{t+1} = I_t + K_t$$

- Preferences

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \cdots$$

- Intertemporal budget constraint

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(A_sF(K_s) - (K_{s+1} - K_s))$$

where we have imposed the transversality condition

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1} = 0$$

- Representative agent maximises

$$\begin{aligned} & E_t \sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s + A_sF(K_s) - (K_{s+1} - K_s) - B_{s+1}) \\ = \; & u(\underbrace{(1+r)B_t + A_tF(K_t) - (K_{t+1} - K_t) - B_{t+1}}_{C_t}) \\ & +\beta E_t \left[u(\underbrace{(1+r)B_{t+1} + A_{t+1}F(K_{t+1}) - (K_{t+2} - K_{t+1}) - B_{t+2}}_{C_{t+1}}) \right] \\ & + \ldots \end{aligned}$$

– First-order condition with respect to B_{t+1}

$$u'(C_t) = \beta(1+r)E_t\left[u'(C_{t+1})\right]$$

– First-order condition with respect to K_{t+1}

$$-u'(C_t) + \beta E_t\left[u'(C_{t+1})\left(A_{t+1}F'(K_{t+1}\right) + 1)\right] = 0 \qquad (*)$$

$$u'(C_t) = \beta E_t\left[u'(C_{t+1})\left(1 + A_{t+1}F'(K_{t+1}\right))\right]$$

$$1 = E_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\left(1 + A_{t+1}F'(K_{t+1}\right))\right]$$

– Recall that if X and Y are random variables then

$$Cov\left[X,Y\right] = E\left[XY\right] - E\left[X\right]E\left[Y\right]$$

and

$$Cov\left[a_0 + X, Y\right] = Cov\left[X,Y\right]$$

– We can rewrite (*) as

$$
\begin{aligned}
1 &= E_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\right] E_t\left[\Big(1 + A_{t+1}F'(K_{t+1})\Big)\right] \\
&\quad + Cov_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right]
\end{aligned}
$$

– From the Euler equation $E_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\right] = \frac{1}{1+r}$

$$
\begin{aligned}
1 &= \frac{1}{1+r} E_t\left[1 + A_{t+1}F'(K_{t+1})\right] \\
&\quad + Cov_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right] \\
1 + r &= E_t\left[1 + A_{t+1}F'(K_{t+1})\right] \\
&\quad + (1+r) Cov_t\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right]
\end{aligned}
$$

– Assuming that $\beta = \frac{1}{1+r}$

$$\begin{aligned} 1 + r &= E_t\left[1 + A_{t+1}F'(K_{t+1})\right] \\ &\quad + Cov_t\left[\frac{u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right] \\ E_t\left[A_{t+1}F'(K_{t+1})\right] &= r - Cov_t\left[\frac{u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right] \end{aligned}$$

* The covariance term is likely to be negative: when the return on investment is high, consumption is likely to be high and hence, the marginal utility of consumption is likely to be low

* Riskiness of capital discourages investment (positive risk premium)

- Certainty equivalence case

$$E_t\left[A_{t+1}F'(K_{t+1})\right] = r$$

  - Productivity shocks affect the date t current account balance via two channels: investment and saving.
  - Sign of current account effect depends on the expected persistence of the productivity shock and the other parameters of the model
  - Graph shows the impulse responses of the current account to a 1% innovation in productivity on date t (assuming $r = 0.05$ and $Y = AK^{0.4}$)

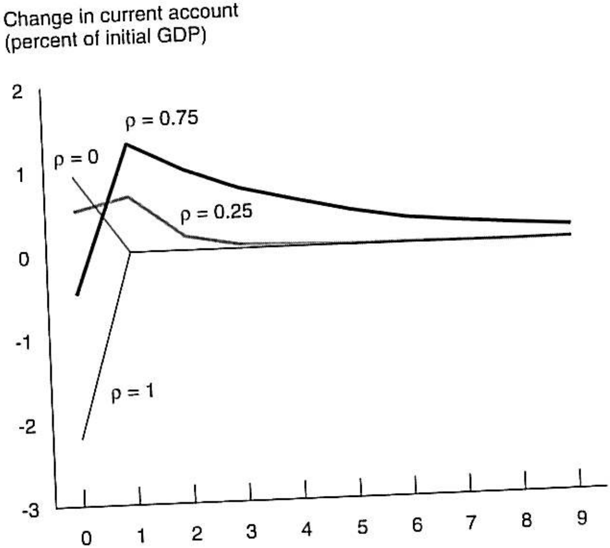


Figure 2.3
Dynamic current-account response to a 1 percent productivity increase